OCL-Lite: A Decidable (Yet Expressive) Fragment of OCL*

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Abstract. UML has become a de facto standard in conceptual modeling. Class diagrams in UML allow one to model the data in the domain of interest by specifying a set of graphical constraints. However, in most cases one needs to provide the class diagram with additional semantics to completely specify the domain, and this is where OCL comes into play. While reasoning over class diagrams is decidable and has been investigated intensively, it is well known that checking the correctness of OCL constraints is undecidable. Thus, we introduce OCL-Lite, a fragment of the full OCL language and prove that reasoning over UML class diagrams with OCL-Lite constraints is in ExpTime by an encoding in the description logic $\mathcal{ALCI}$. As a side result, DL techniques and tools can be used to reason on UML class diagrams annotated with arbitrary OCL-Lite constraints.

1 Introduction

The Unified Modeling Language (UML) has become a de facto standard in conceptual modeling of information systems. In UML, a conceptual schema is represented by means of a class diagram, with its graphical constraints, together with a set of user-defined constraints, which are usually specified in the Object Constraint Language (OCL). In every application domain the set of instances that can be stored or managed is necessarily finite and, thus, a schema has not only to be consistent, but also finitely satisfiable [13]. It is well-known that reasoning with OCL integrity constraints in their full generality is undecidable since it amounts to full FOL reasoning [4]. Thus, reasoning with UML conceptual schemas in the presence of OCL constraints has been approached in the following alternative ways:

1. Allowing general constraints (in OCL or other languages) without guaranteeing termination in all cases [9, 6, 15].

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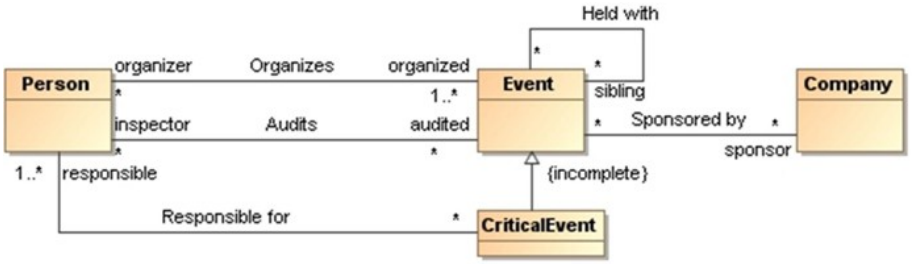

Integrity constraints

1. **context** Person **inv:** organized->exists(oclIsTypeOf(CriticalEvent) and sponsor->isEmpty())
2. **context** Event **inv:** sibling->notEmpty() implies not oclIsTypeOf(CriticalEvent)
3. **context** Person **inv:** audited->forall(sponsor->notEmpty())
4. **context** CriticalEvent **inv:** not responsible.organized-> isEmpty()
5. **context** Company **inv:** event->select(oclIsTypeOf(CriticalEvent))->notEmpty() or event->exists(sibling->isEmpty() or sibling->select(sponsor->isEmpty())-> notEmpty())

Fig. 1. UML class diagram with OCL constraints

2. Allowing general constraints and ensuring termination without guaranteeing completeness of the result [1, 17, 12].
3. Ensuring both termination and completeness of finite reasoning by allowing only specific kinds of constraints [13, 11, 18].
4. Ensuring terminating and complete reasoning by disallowing OCL constraints and admitting unrestricted models [8, 5, 10, 3, 2].

To our knowledge, none of the existing approaches guarantees complete and terminating reasoning on UML schemas coupled with OCL constraints able to capture those in Figure 1. With approaches of the first kind, a result may not be obtained in some particular cases. On the contrary, the second kind of approaches may fail to find existing valid solutions. Approaches of the third kind do not allow arbitrary constraints as the ones in Figure 1. Finally, the last approaches are based on a Description Logic (DL) encoding of a UML schema. These methods do not consider OCL constraints and they usually check unrestricted satisfiability.

The main purpose of this paper is to identify a fragment of OCL, which we call OCL-Lite, that guarantees *finite satisfiability* while being significantly expressive at the same time. In other words, we propose the specification of arbitrary constraints within the bounds of OCL-Lite in a UML conceptual schema to ensure completeness and decidability of reasoning. Such nice properties are due to the *finite model property* (FMP) of the language, i.e., it is guaranteed that a satisfiable UML/OCL-Lite schema is always finitely satisfiable. The proposed OCL-Lite is the result of identifying a fragment of OCL that can be encoded into the DL $\mathcal{ALCI}$ [7], which has interesting reasoning properties. In particular, $\mathcal{ALCI}$ enjoys the FMP, i.e., every satisfiable set of constraints formalized in

$\mathcal{ALCI}$ admits a finite model. Thus, the FMP is also guaranteed for any fragment of OCL that fits into $\mathcal{ALCI}$.

The contributions of this paper can be summarized as follows:

- The identification of a fragment of OCL that enjoys the FMP. To our knowledge, the reasoning properties of OCL had not been studied before, except that full OCL leads to undecidability.
- A mapping from OCL-Lite to the DL $\mathcal{ALCI}$ to prove that the proposed fragment has the same reasoning properties as $\mathcal{ALCI}$. To our knowledge, this is the first attempt to encode OCL constraints in DLs. As a side result, a DL reasoner can be used to verify the correctness of a schema.
- Thanks to the mapping to $\mathcal{ALCI}$ we are able to show both the FMP and that checking satisfiability in UML/OCL-Lite is an ExpTime-complete problem.

2 OCL-Lite Syntax and Semantics

In this section we present the fragment of OCL that corresponds to OCL-Lite. We start by an overview of basic concepts of UML and OCL.

A UML class diagram represents the static view of the domain basically by means of classes and associations between them, representing, respectively, sets of objects and relations between objects. An association is constrained by a set of participating classes. An *association end* is a part of an association that defines the participation of a class in the association. The name of the *role* played by a participant in an association is placed in the association end near the corresponding class. If the role name is omitted, the role played by the participant is the name of its class.

An *OCL constraint* (or invariant) has the form: `context` *`C`* `inv:` *`OCLExpr`*, where *`C`* is the *contextual class*, i.e., the class to which the constraint belongs, and *`OCLExpr`* is an expression that results in a boolean value. An OCL constraint is satisfied by an instantiation of the schema if *`OCLExpr`* evaluates to true for each instance of the contextual class. An *OCL expression* is defined by means of navigation paths, combined with predefined OCL operations to specify conditions on those paths. A *navigation path* is a sequence of role names defined in the associations of the class diagram. Each role name used in a path indicates the direction of the navigation.

2.1 OCL-Lite Syntax

In this section we specify the syntax rules that allow one to construct OCL constraints belonging to the fragment of OCL that we call OCL-Lite. An *OCL-Lite constraint* has the form: `context` *`C`* `inv:` *`OCL-LiteExpr`*. In the syntax rules, shown in Figure 2, an *OCL-Lite expression* *`OCL-LiteExpr`* is defined recursively. Intuitively, *`OCL-LiteExpr`* allows one to construct a boolean OCL-Lite expression, which can correspond to the whole constraint, or can be the condition specified as a parameter in a `select` operation (see *`SelectExpr`*) or in `exists`

```
OCL-LiteExpr ::= Path SelectExpr | oclIsTypeOf(C) | not OCL-LiteExpr |
                 OCL-LiteExpr and OCL-LiteExpr |
                 OCL-LiteExpr or OCL-LiteExpr |
                 OCL-LiteExpr implies OCL-LiteExpr
Path         ::= PathItem | PathItem.Path
PathItem     ::= r_i | oclAsType(C).r_i
SelectExpr   ::= BooleanOp | ->select(OCL-LiteExpr) SelectExpr
BooleanOp    ::= ->exists(OCL-LiteExpr) | ->forall(OCL-LiteExpr) |
                 ->size()>0 | ->size()=0 | ->isEmpty() | ->notEmpty()
```

Fig. 2. Syntax of OCL-Lite expressions

and `forall` operations (see *BooleanOp*). An *OCL-LiteExpr* can be either a *Path* to which a *SelectExpr* is applied, a check of whether an object is of a certain type, or boolean combinations of these OCL-Lite expressions.

The label *Path* indicates how a navigation path can be constructed as a non-empty sequence of *PathItem*s. Each *PathItem* can be either a role name r_i specified in the class diagram, or a role name preceded by the operation *oclAsType(C)*, when we need to access a role name of a particular class *C*. When *OCL-LiteExpr* corresponds to the whole constraint, each path starts from a role that is accessible from the contextual class (or a subclass *C* of the contextual class, in which case `oclAsType(`*C*`)` must be specified first). Otherwise, when *OCL-LiteExpr* is inside a `select`, `exists`, or `forall` operation, then, the starting role name will depend on the context where the operation is used. After a *Path*, one can apply a (possibly empty) sequence of selections on the collection of objects obtained through the path, always followed by a *BooleanOp*.

The intuitive meaning of the OCL collection operations included in this fragment of OCL is as follows. Let *col* denote the collection of objects reachable along a path, and let *o* denote a single object, then:

- *col*`->select(`*OCL-LiteExpr*`)`: returns the subset of elements of *col* that satisfy *OCL-LiteExpr*;
- *col*`->exists(`*OCL-LiteExpr*`)`: returns true iff there is some element of *col* that satisfies *OCL-LiteExpr*;
- *col*`->forall(`*OCL-LiteExpr*`)`: returns true iff all the elements of *col* satisfy *OCL-LiteExpr*;
- *col*`->size()`: returns the number of elements of *col*;
- *col*`->isEmpty()`: returns true iff *col* is empty;
- *col*`->notEmpty()`: returns true iff *col* is not empty;
- *o*`.oclIsTypeOf(`*C*`)`: returns true iff *o* is an instance of the class *C*;

All constraints in Figure 1 are examples of well-formed OCL-Lite expressions.

2.2 OCL-Lite Semantics

OCL-Lite operations, except for `oclIsTypeOf` and `oclAsType`, can be expressed only in terms of `select`, `isEmpty`, and `notEmpty`. Thus, to specify the semantics of OCL-Lite expressions, we first rewrite each expression as an equivalent

Table 1. Normalization of OCL-Lite expressions

	Original expression	**Normalized expression**
a)	`col->exists(cond)`	`col->select(cond)->notEmpty()`
b)	`col->forall(cond)`	`col->select(not cond)->isEmpty()`
c)	`col->select(cond1)->select(cond2)`	`col->select(cond1 and cond2)`
d)	`col->size()>0`	`col->notEmpty()`
e)	`col->size()=0`	`col->isEmpty()`
f)	`not col->isEmpty()`	`col->notEmpty()`
g)	`not col->notEmpty()`	`col->isEmpty()`

normalized one, which is expressed in terms of these operations only. Table 1 shows the OCL-Lite expressions together with their normal form. These normalizations, together with de Morgan's laws, are iteratively applied until the expression only contains the operations `select`, `isEmpty`, and `notEmpty`, and the boolean operator `not` only appears before `oclIsTypeOf(C)`. In the table, `col` and `cond` denote, respectively, a collection and a condition, which must be defined according to the syntax rules.

In our running example, the constraints in Figure 1 that have to be normalized are 1, 3, 4, and 5. The resulting expressions we get after applying the rules in Table 1 are:

- Constraint 1. We apply rule a) and we get the normalized expression:
```
context Person inv:
   organized->select(oclIsTypeOf(CriticalEvent) and
                     sponsor->isEmpty())->notEmpty()
```
- Constraint 3. We first apply rule b) and we get:
```
context Person inv:
   audited->select(not sponsor->notEmpty())->isEmpty()
```
  We then apply rule g) to obtain the normalized expression:
```
context Person inv:
   audited->select(sponsor->isEmpty())->isEmpty()
```
- Constraint 4. We apply rule f) and we get:
```
context CriticalEvent inv: responsible.organized->notEmpty()
```
- Constraint 5. We apply rule a) and we get:
```
context Sponsor inv:
   event->select(oclIsTypeOf(CriticalEvent))->notEmpty() or
   event->select(sibling->isEmpty() or
     sibling->select(sponsor->isEmpty())->notEmpty())->notEmpty()
```

It can be seen from Table 1 that the expressions resulting from the normalization conform to a limited set of patterns, basically consisting of an optional `select` operation followed either by `isEmpty` or `notEmpty`. Also, the expression may include the operation `oclIsTypeOf`.

In the following we consider OCL-Lite expressions in their normal form. For each of them we specify its semantics by means of an interpretation function, f,

that maps each *OCL-LiteExpr* into a FOL formula $\mathit{OCL\text{-}LiteExpr}^f(x)$ with one free variable. Other approaches specify the semantics of OCL expressions using first-order terms instead of formulas [4]. However, as also argued in [4], using formulas is preferable when dealing with sets, as in our case.

We start by formalizing the semantics of an OCL-Lite constraint

`context` *C* `inv:` *OCL-LiteExpr*

Its interpretation is: $\forall x\,(C(x) \rightarrow \mathit{OCL\text{-}LiteExpr}^f(x))$ where C is the unary predicate corresponding to class *C*.

To define the semantics of OCL-Lite expressions, we first introduce some notation to deal with navigation paths. Consider a navigation path $p_n \ldots p_1$ in an OCL-Lite expression, where each p_i is either a role name or `oclAsType(`C_i`).`r_i. To formalize a (binary) association A_i, we introduce a binary predicate, A_i, whose first argument represents an instance of $dom(A_i)$ (the domain of A_i) and whose second argument represents an instance of $range(A_i)$ (the range of A_i). To fix a semantics for role names we conform to the UML convention about role names [16], i.e., a role name attached to an association end labeled with a class *C* is used to navigate from one object to another one belonging to the class *C*. Thus, in the following, $p_i^f(x,y) = A_i(x,y)$ when p_i is a role name attached to the $range(A_i)$-end of A_i, and, viceversa, $p_i^f(x,y) = A_i(y,x)$ when p_i is a role name attached to the $dom(A_i)$-end of A_i. Similarly, $p_i^f(x,y) = C_i(x) \wedge A_i(x,y)$, when $p_i =$ `oclAsType(`C_i`).`r_i and r_i is a role name attached to the $range(A_i)$-end of A_i, while $p_i^f(x,y) = C_i(x) \wedge A_i(y,x)$, when $p_i =$ `oclAsType(`C_i`).`r_i and r_i is a role name attached to the $dom(A_i)$-end of A_i.

1. *OCL-LiteExpr* $= p_n \ldots p_1$`->select(`*OCL-LiteExpr*$_0$`)->notEmpty()`
   The semantics of this expression is

   $$\mathit{OCL\text{-}LiteExpr}^f(x) = \exists x_n \cdots \exists x_1 (p_n^f(x,x_n) \wedge \cdots \wedge p_1^f(x_2,x_1) \wedge \mathit{OCL\text{-}LiteExpr}_0^f(x_1))$$

   A particular case of this expression is when no `select` operation is applied on the objects obtained from the navigation, which corresponds to the expression *OCL-LiteExpr* $= p_n \ldots p_1$`->notEmpty()`. In this case, we have

   $$\mathit{OCL\text{-}LiteExpr}^f(x) = \exists x_n \cdots \exists x_1\, (p_n^f(x,x_n) \wedge \cdots \wedge p_1^f(x_2,x_1))$$
2. *OCL-LiteExpr* $= p_n \ldots p_1$`->select(`*OCL-LiteExpr*$_0$`)->isEmpty()`
   The semantics of this expression is

   $$\mathit{OCL\text{-}LiteExpr}^f(x) = \forall x_n \cdots \forall x_1 (\neg p_n^f(x,x_n) \vee \cdots \vee \neg p_1^f(x_2,x_1) \vee \neg \mathit{OCL\text{-}LiteExpr}_0^f(x_1))$$

   Again, we have a particular case of this kind of expression in the absence of `select`, namely *OCL-LiteExpr* $= p_n \ldots p_1$`->isEmpty()`. In this case, the semantics of the expression is

   $$\mathit{OCL\text{-}LiteExpr}^f(x) = \forall x_n \cdots \forall x_1\, (\neg p_n^f(x,x_n) \vee \cdots \vee \neg p_1^f(x_2,x_1))$$
3. *OCL-LiteExpr* $=$ [`not`] `oclIsTypeOf(`*C*`)`
   where the brackets denote optionality. The semantics of the expression is

   $$\mathit{OCL\text{-}LiteExpr}^f(x) = [\neg] C(x)$$

4. *OCL-LiteExpr* = *OCL-LiteExpr*$_1$ **and** *OCL-LiteExpr*$_2$
   The semantics of this expression is

$$\mathit{OCL\text{-}LiteExpr}^f(x) = \mathit{OCL\text{-}LiteExpr}_1^f(x) \wedge \mathit{OCL\text{-}LiteExpr}_2^f(x)$$

5. *OCL-LiteExpr* = *OCL-LiteExpr*$_1$ `or` *OCL-LiteExpr*$_2$
   The semantics of this expression is

$$\mathit{OCL\text{-}LiteExpr}^f(x) = \mathit{OCL\text{-}LiteExpr}_1^f(x) \vee \mathit{OCL\text{-}LiteExpr}_2^f(x)$$

6. *OCL-LiteExpr* = *OCL-LiteExpr*$_1$ `implies` *OCL-LiteExpr*$_2$
   The semantics of this expression is

$$\mathit{OCL\text{-}LiteExpr}^f(x) = \mathit{OCL\text{-}LiteExpr}_1^f(x) \rightarrow \mathit{OCL\text{-}LiteExpr}_2^f(x)$$

Definition 1 (Satisfiability of OCL-Lite constraints). *Let Γ be a set of OCL-Lite constraints and Γ^f the resulting FOL theory. Then, Γ is said to be* satisfiable *if there exists a first order interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ that satisfies Γ^f. $\mathcal{I}$ is called a* model *of Γ.*

3 Encoding UML/OCL-Lite in $\mathcal{ALCI}$

In this section we show that the proposed fragments of OCL and UML can both be encoded in the DL $\mathcal{ALCI}$. Thus, finite reasoning is guaranteed for every conceptual schema expressed in this language. We first present the fragment of UML we are interested in, and then we provide an encoding for the OCL-Lite fragment, too.

We assume the encoding of a fragment of UML class diagrams in $\mathcal{ALCI}$, based on the encoding in $\mathcal{ALCQI}$ proposed in [5]. Since $\mathcal{ALCQI}$ is an extension of $\mathcal{ALCI}$ that does not have the FMP [7] (i.e., a schema specified in $\mathcal{ALCQI}$ might be satisfiable only by an infinite number of instances), we focus on a fragment of UML that can be encoded into $\mathcal{ALCI}$. In particular, we consider UML class diagrams where the domain of interest is represented through *classes* (representing sets of objects), possibly equipped with *attributes* and *associations* (representing relations among objects), and *types* (representing the domains of attributes, i.e., integer, string, etc.). The kind of UML constraints that we consider in this paper are the ones typically used in conceptual modeling, namely *hierarchical* relations between classes, *disjointness* and *covering* between classes, *cardinality* constraints for participation of entities in relationships, and *multiplicity* and *typing* constraints for attributes. To preserve the FMP we restrict both cardinality and multiplicity constraints to be of the form `*` or `1..*` (meaning that either no constraint applies or the class participates at-least once, respectively).

The encoding, starting from a UML class diagram Σ, generates a satisfiability preserving $\mathcal{ALCI}$ knowledge base $\mathcal{K}_\Sigma$ (see [5] for details on the encoding). Given the UML fragment chosen, a model of the knowledge base $\mathcal{K}_\Sigma$ can be viewed as an instantiation of the UML class diagram Σ.

In the following, we provide a mapping to translate OCL-Lite constraints into $\mathcal{ALCI}$. An OCL-Lite constraint, which has the general form

$$\texttt{context } C \texttt{ inv: } \textit{OCL-LiteExpr}$$

is encoded as the following $\mathcal{ALCI}$ inclusion assertion:

$$C \sqsubseteq \textit{OCL-LiteExpr}^{\dagger}$$

where $\cdot^{\dagger}$ is a mapping function that assigns to each OCL-Lite expression its corresponding $\mathcal{ALCI}$ encoding. This inclusion assertion, according to the semantics of OCL constraints, states that each instance of C must satisfy $\textit{OCL-LiteExpr}$. We now illustrate the encoding of OCL-Lite expressions in $\mathcal{ALCI}$. We consider OCL-Lite expressions in their normal form, as provided in Section 2.2, and define $\textit{OCL-LiteExpr}^{\dagger}$ by induction on the structure of $\textit{OCL-LiteExpr}$.

1. $\textit{OCL-LiteExpr} = p_n \ldots p_1\texttt{->select(}\textit{OCL-LiteExpr}_0\texttt{)->notEmpty()}$
   We define the $\mathcal{ALCI}$ concept $\textit{OCL-LiteExpr}^{\dagger}$ by induction on the length n of the navigation path. For convenience, we consider as base case $n = 0$, and in this case we set $\textit{OCL-LiteExpr}^{\dagger} = \textit{OCL-LiteExpr}_0^{\dagger}$.
   For the inductive case, let $\textit{OCL-LiteExpr}_n = p_n \ldots p_1\texttt{->select(}\textit{OCL-LiteExpr}_0\texttt{)->notEmpty()}$, let p_{n+1} be an additional path item, and let $\textit{OCL-LiteExpr}_{n+1} = p_{n+1}.\textit{OCL-LiteExpr}_n$. Then $\textit{OCL-LiteExpr}_{n+1}^{\dagger} = p_{n+1}^{\dagger}.(\textit{OCL-LiteExpr}_n^{\dagger})$, where $p_{n+1}^{\dagger}$ (for the various cases of p_{n+1}, cf. the OCL syntax in Section 2.1) is an abbreviation[1] defined as follows (r denotes a role name of an association A, and $dom(A)$ and $range(A)$ denote respectively the domain and range of A):

$$r^{\dagger} = \begin{cases} \exists A & \text{if } r \text{ is attached to } range(A) \\ \exists A^{-} & \text{if } r \text{ is attached to } dom(A) \end{cases}$$

$$(\texttt{oclAsType(}C\texttt{).}r)^{\dagger} = \begin{cases} C \sqcap \exists A & \text{if } r \text{ is attached to } range(A) \\ C \sqcap \exists A^{-} & \text{if } r \text{ is attached to } dom(A) \end{cases}$$

   Note that the $\mathcal{ALCI}$ concept corresponding to $\textit{OCL-LiteExpr}$ has the form

$$p_n^{\dagger}.(p_{n-1}^{\dagger}.(\cdots(p_1^{\dagger}.(\textit{OCL-LiteExpr}_0^{\dagger}))\cdots))$$

   Intuitively, this concept represents the fact that $\textit{OCL-LiteExpr}$ evaluates to true, for a given instance o in the domain of p_n, if o is related through the path $p_n \ldots p_1$ to some object o_1 that satisfies the condition specified by $\textit{OCL-LiteExpr}_0$. When there is no `select` operation, i.e., the OCL expression has the form $p_n \ldots p_1\texttt{->notEmpty()}$, then $\textit{OCL-LiteExpr}_0^{\dagger} = \top$, and the constraint is encoded as $p_n^{\dagger}.(p_{n-1}^{\dagger}.(\cdots\ (p_1^{\dagger}.\top)\cdots))$.
   For example, once it has been normalized in Section 2.2, an expression that follows this pattern is the one in the body of constraint 4. The $\mathcal{ALCI}$ assertion that encodes this constraint is:

$$\mathsf{CriticalEvent} \sqsubseteq \exists \mathsf{ResponsibleFor}^{-}.(\exists \mathsf{Organizes}.\top)$$

[1] Note that $p^{\dagger}$ is not a valid DL expression.

Note that, the first DL role, $\mathsf{ResponsibleFor}^-$, is inverted since the association `ResponsibleFor` has domain `Person` and range `CriticalEvent`, and the role name `responsible` is attached to `Person`. Thus, $\texttt{responsible}^\dagger = \exists\mathsf{ResponsibleFor}^-$. The next role name in the OCL-Lite expression is `organized`, which is attached to `Event`, the range of the association `Organizes`. Thus, $\texttt{organized}^\dagger = \exists\mathsf{Organizes}$. Finally, since the OCL operation `select` does not appear in the constraint, no condition must be imposed on the instances reached at the end of the path.

2. $\mathit{OCL\text{-}LiteExpr} = p_n \ldots p_1\texttt{->select(}\mathit{OCL\text{-}LiteExpr}_0\texttt{)->isEmpty()}$
Similarly to the previous case, we define $\mathit{OCL\text{-}LiteExpr}^\dagger$ by induction on n. For the base case of $n = 0$, we set $\mathit{OCL\text{-}LiteExpr}^\dagger = \neg\mathit{OCL\text{-}LiteExpr}_0^\dagger$. For the inductive case, let:
$\mathit{OCL\text{-}LiteExpr}_n = p_n \ldots p_1\texttt{->select(}\mathit{OCL\text{-}LiteExpr}_0\texttt{)->isEmpty()}$,
let p_{n+1} be an additional path item, and let
$\mathit{OCL\text{-}LiteExpr}_{n+1} = p_{n+1}.\mathit{OCL\text{-}LiteExpr}_n$.
Then $\mathit{OCL\text{-}LiteExpr}_{n+1}^\dagger = \neg p_n^\dagger.(\neg\mathit{OCL\text{-}LiteExpr}_n^\dagger)$,
Considering that $\neg\neg C$ is equivalent to C, the $\mathcal{ALCI}$ concept corresponding to $\mathit{OCL\text{-}LiteExpr}$ has the form

$$\neg(p_n^\dagger.(p_{n-1}^\dagger.(\cdots(p_1^\dagger.(\mathit{OCL\text{-}LiteExpr}_0^\dagger))\cdots)))$$

Intuitively, this concept represents the fact that $\mathit{OCL\text{-}LiteExpr}$ evaluates to true, for a given instance o, if o is not related through the path $p_n \ldots p_1$ to any object that satisfies the condition specified by $\mathit{OCL\text{-}LiteExpr}_0$. As in the previous case, if there is no `select` operation in the OCL expression, i.e., the OCL expression has the form $p_n \ldots p_1\texttt{->isEmpty()}$, then $\mathit{OCL\text{-}LiteExpr}_0^\dagger = \top$, and the constraint is encoded as $\neg(p_n^\dagger.(p_{n-1}^\dagger.(\cdots\ (p_1^\dagger.\top)\cdots)))$
As an example, let us consider constraint 3 in its normal form. The overall OCL-Lite expression is encoded in $\mathcal{ALCI}$ as $\neg(\exists\mathsf{Audits}.(\neg\exists\mathsf{SponsoredBy}.\top))$, and the $\mathcal{ALCI}$ assertion corresponding to the constraint is:

$$\mathsf{Person} \sqsubseteq \neg\exists\mathsf{Audits}.\neg\exists\mathsf{SponsoredBy}.\top$$

3. $\mathit{OCL\text{-}LiteExpr} = \texttt{oclIsTypeOf(}C\texttt{)}$, $\mathit{OCL\text{-}LiteExpr} = \texttt{not oclIsTypeOf(}C\texttt{)}$
The $\mathcal{ALCI}$ concept $\mathit{OCL\text{-}LiteExpr}^\dagger$ corresponding to these OCL-Lite expressions is respectively

$$C, \qquad \text{and} \qquad \neg C,$$

4. $\mathit{OCL\text{-}LiteExpr} = \mathit{OCL\text{-}LiteExpr}_1\ \texttt{and}\ \mathit{OCL\text{-}LiteExpr}_2$
The corresponding $\mathcal{ALCI}$ concept $\mathit{OCL\text{-}LiteExpr}^\dagger$ is

$$\mathit{OCL\text{-}LiteExpr}_1^\dagger \sqcap \mathit{OCL\text{-}LiteExpr}_2^\dagger$$

5. $\mathit{OCL\text{-}LiteExpr} = \mathit{OCL\text{-}LiteExpr}_1\ \texttt{or}\ \mathit{OCL\text{-}LiteExpr}_2$
The corresponding $\mathcal{ALCI}$ concept $\mathit{OCL\text{-}LiteExpr}^\dagger$ is

$$\mathit{OCL\text{-}LiteExpr}_1^\dagger \sqcup \mathit{OCL\text{-}LiteExpr}_2^\dagger$$

6. $\mathit{OCL\text{-}LiteExpr} = \mathit{OCL\text{-}LiteExpr}_1$ `implies` $\mathit{OCL\text{-}LiteExpr}_2$
   The corresponding $\mathcal{ALCI}$ concept $\mathit{OCL\text{-}LiteExpr}^\dagger$ is

$$\neg \mathit{OCL\text{-}LiteExpr}_1^\dagger \sqcup \mathit{OCL\text{-}LiteExpr}_2^\dagger$$

To further illustrate the mapping, we apply it to some other constraints of our running example. For instance, consider the normalized expression of constraint 1. This constraint is of kind 1, and its subexpressions are respectively of kinds 4, 3, and 2. Applying the corresponding mapping rules we obtain:

$$\mathsf{Person} \sqsubseteq \exists \mathsf{Organizes}.(\mathsf{CriticalEvent} \sqcap \neg \exists \mathsf{SponsoredBy}.\top)$$

As an example of an OCL-Lite expression of kind 6 we have constraint 2. According to the mapping rule, its corresponding $\mathcal{ALCI}$ expression is:

$$\mathsf{Event} \sqsubseteq \neg \exists \mathsf{HeldWith}.\top \sqcup \neg \mathsf{CriticalEvent}$$

Constraint 5 is of kind 5 once normalized. We recursively apply the mapping rules to each part of the disjunction: rules 1 and 3 to the first subexpression, and rules 5, 2, 1 to the second subexpression. The resulting $\mathcal{ALCI}$ expression is:

$$\mathsf{Company} \sqsubseteq \exists \mathsf{SponsoredBy}^-.\mathsf{CriticalEvent} \sqcup \exists \mathsf{SponsoredBy}^-.(\neg \exists \mathsf{HeldWith}.\top \sqcup \exists \mathsf{HeldWith}.\neg \exists \mathsf{SponsoredBy}.\top)$$

The following theorem states that the mapping from OCL-Lite to $\mathcal{ALCI}$ is correct (we refer to [14] for the proof).

Theorem 1 (Correctness of the OCL-Lite encoding). *Let Γ be a set of OCL-Lite constraints and $\mathcal{K}_\Gamma$ its $\mathcal{ALCI}$ encoding. Then, Γ is satisfiable if and only if $\mathcal{K}_\Gamma$ is satisfiable.*

Concerning the complexity of reasoning over UML/OCL-Lite schemas we first notice that reasoning just over the UML diagrams as proposed in this paper is an NP-complete problem. Indeed, the UML language we consider here is a sub-language of ER_{bool} which was proved to be NP-complete in [3], and the very same complexity proof applies to the UML language we use.

Theorem 2 (Complexity of UML/OCL-Lite). *Checking the satisfiability of UML/OCL-Lite conceptual schemas is an* ExpTime*-complete problem.*

The upper bound follows from the fact that the $\mathcal{ALCI}$ encoding is correct, and that reasoning in $\mathcal{ALCI}$ is in ExpTime. The lower bound is established by reducing satisfiability of $\mathcal{ALC}$ TBoxes, which is known to be ExpTime-complete, to satisfiability of OCL-Lite constraints (see [14] for the proof).

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