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INTERIOR MODE SELECTION IN THE CRAIG BAMPTON REDUCTION TECHNIQUE BASED ON AN ENERGY APPROACH

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ABSTRACT

Finite element analysis is adopted in several engineering fields as such structural dynamics and multibody system dynamics. Fine meshes are usually employed to guarantee model accuracy. As a consequence, models have typically very large dimensions, and hence they are difficult to handle and often prone to numerical ill conditioning. Model reduction techniques, such as the Craig Bampton method, can be extremely useful to minimize model dimensions.

This paper introduces an effective ranking method for the selection of CB interior modes suitable for vibrating systems under single harmonic excitation. The goal is to keep model dimensions to a minimum while preserving system forced response accuracy. The ranking of the interior modes is carried out using coefficients based on energetic considerations and taking into account the frequency and the shape of the force exciting the system. The aforementioned coefficients provide a measure of the contribution of each interior mode to the computation of the mean mechanical energy stored by the system in a period of excitation.

The method is then applied to the model of a vibratory feeder. The results shows that the proposed method provides a very effective selection of the most important interior modes and therefore allows outperforming current state-of-the-art techniques.

Keywords: Finite element models, Craig Bampton method, Mode selection, Model reduction

1. INTRODUCTION

1.1. General information

Accurate finite element models are very useful in several highly technological engineering fields such as structural dynamics, multibody system dynamics, control engineering and signal processing. Unfortunately, accurate models have typically large dimensions, which may pose a limit to their

usefulness for example for simulation [1], control design [2], optimization techniques [3, 4], model parameter identification [5]. In order to overcome such a problem several model reduction techniques have been developed in the last decades. In the structural dynamics and multibody fields, one of the most widespread reduction techniques is the Craig–Bampton (CB) method [6], in consequence of its straightforwardness. The Craig Bampton method has been developed for systems which can be described through linear FE representations. Basically this method is a combination of Guyan's condensation and modal truncation. As Guyan's condensation, it uses the static deflection shapes of some nodes of the system, called master nodes, and enriches this space with a reduced set of interior vibrational modes of the system to increase the accuracy.

The practical implementation of the CB method imposes partitioning the physical coordinates $\boldsymbol{x} \in \mathbb{R}^n$ into a subset of m master degrees of freedom (dofs) $\boldsymbol{x}_\mathrm{e} \in \mathbb{R}^m$, usually referred to as external dofs, and a subset of s slave dofs $\boldsymbol{x}_\mathrm{i} \in \mathbb{R}^s$, the so called interior dofs, with $m + s = n$:

$$\boldsymbol{x}(t) = \{\boldsymbol{x}_\mathrm{e}^\mathrm{T} \quad \boldsymbol{x}_\mathrm{i}^\mathrm{T}\}^\mathrm{T} \tag{1}$$

Typically, such a distinction is made selecting as master dofs those on which external loads are applied or those lying at the interface with other subsystems, or, more in general, those that are of some interest (e.g. for measurements, monitoring and so on). Indeed, the master dofs are entirely retained in the reduced model.

Then, the coordinate set is transformed into a hybrid coordinate set $\boldsymbol{y}$ by means of the non-singular CB transformation matrix $\mathbf{H} \in \mathbb{R}^{n \times n}$:

$$\boldsymbol{x}(t) = \begin{Bmatrix} \boldsymbol{x}_\mathrm{e}(t) \\ \boldsymbol{x}_\mathrm{i}(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{I}_\mathrm{m} & \mathbf{0} \\ \mathbf{B} & \boldsymbol{\Phi} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_\mathrm{e}(t) \\ \boldsymbol{\eta}(t) \end{Bmatrix} = \mathbf{H}\boldsymbol{y}(t) \tag{2}$$

The new hybrid coordinate set includes the master dofs $\boldsymbol{x}_\mathrm{e}$, and the interior modal coordinates $\boldsymbol{\eta}$, which are the modal coordinates related to the interior vibrational mode eigenvectors $\boldsymbol{\phi}_\varsigma \in \mathbb{R}^s$ $(\varsigma = 1, \dots, \mathrm{s})$, i.e. the vibrational modes of the system obtained by constraining the set of master dofs. In Eq. (2) $\mathbf{B} \in \mathbb{R}^{s \times m}$ is a Guyan's reduction basis, and $\boldsymbol{\Phi} \in \mathbb{R}^{s \times s}$ is the eigenvector matrix, i.e. the matrix whose columns are eigenvectors $\boldsymbol{\phi}_\varsigma$ $(\varsigma = 1, \dots, \mathrm{s})$. Finally, $\mathbf{I}_\mathrm{p} \in \mathbb{R}^{p \times p}$ and $\mathbf{0} \in \mathbb{R}^{p \times q}$ represent, respectively, the identity and the null matrices (for any arbitrary integer scalar p and q).

In order to reduce model dimensions, the interior modal coordinate vector $\boldsymbol{\eta}$ is truncated to a smaller vector $\tilde{\boldsymbol{\eta}} \in \mathbb{R}^r$ $(r \ll s)$:

$$\boldsymbol{x}(t) \cong \begin{bmatrix} \mathbf{I}_\mathrm{m} & \mathbf{0} \\ \mathbf{B} & \tilde{\boldsymbol{\Phi}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_\mathrm{e}(t) \\ \tilde{\boldsymbol{\eta}}(t) \end{Bmatrix} = \tilde{\mathbf{H}}\tilde{\boldsymbol{y}}(t) \tag{3}$$

where $\tilde{\mathbf{H}} \in \mathbb{R}^{n \times (m+r)}$ is the CB reduction matrix, obtained from Eq. (2) by removing the columns of $\boldsymbol{\Phi}$ associated to the interior modes which are neglected.

A crucial aspect in the practical implementation of such a method is the proper selection of the reduced set of interior modes to be retained in reduced models to keep the dimensions to a minimum while guaranteeing a satisfactory accuracy. The most used approach to select the CB interior modes consists in retaining only the interior modes with the lowest eigenfrequencies [7] (typically up to two times the highest frequency of interest). Following this approach, unfortunately, high frequency interior modes, whose participation in the system dynamics may be considerable, could be discarded and low frequency modes, whose contribution may be negligible, could be retained in the reduced model. Therefore, this criterion to select the interior modes, though simple, is far from being optimal. In order to perform a more effective reduction using the CB method, some methods have been proposed in literature to rank and select the interior modes. Among the ranking methods proposed in literature one should at least recall the "Component Mode Synthesis χ" (CMS χ) [8], the "Effective Interface Mass" (EIM) [9, 10], the "Interior Mode Ranking" (IMR) [11] and the "Optimal Modal Reduction" (OMR) [12, 13]. Basically, the CMS χ, the EIM and the OMR are general purpose

methods based on some terms representing the coupling between the interior modes and the system interface. Unfortunately, they all overlook the frequency range at which reduced order models should be accurate, as well as the frequencies and the spatial distribution of the external force acting on the system, even if they are known. Conversely, the IMR method takes into account the frequencies of interest, but neglects the spatial distribution of the external force, moreover it has been developed just for the reduction of resonant systems.

Lots of systems are designed to operate excited with a know external force at specific frequencies, as for example vibratory feeders and sieves. In these cases, a more accurate and efficient model reduction can be performed taking into account the actual forces acting on a system. This evidence justifies the development of a novel ranking method, introduced in this paper, for the selection of the CB interior modes, and suitable for vibrating systems under harmonic excitation. The method has been finalized in order to rank and select the minimum number of the interior modes ensuring the achievement of an accurate representation of the system forced response.

The underlying idea is that the most important interior modes are those that provide the largest energy contributions to the system forced response. The contribution of each interior mode to the mean mechanical energy stored by the system in a period of excitation is evaluated by means of scalar coefficients analytically defined.

The paper outline is the follows: in Section 2 the proposed ranking method is introduced and discussed. In Section 3 the method is applied to a linear vibratory feeder. The effectiveness of the method in achieving an accurate representation of the system forced response is proved and compared with the one of the aforementioned state-of-the-art methods. Concluding remarks are given in the Section 4.

2. RANKING METHOD

Let us consider a *n*-dimensional linear time-invariant and undamped vibrating system, represented through its stiffness and mass matrices $\mathbf{K}, \mathbf{M} \in \mathbb{R}^{n\times n}$ and the physical coordinate vector $\boldsymbol{x}$. Let the system be excited on the master dofs by a known harmonic force $\boldsymbol{f}$ at angular frequency ω:

$$\boldsymbol{f}(t) = \begin{Bmatrix} \boldsymbol{f}_{\mathrm{e}}(t) \\ \mathbf{0} \end{Bmatrix} = \begin{Bmatrix} \mathrm{f}_1 \cos(\omega t + \alpha_1) \\ \vdots \\ \mathrm{f}_m \cos(\omega t + \alpha_m) \\ 0_{m+1} \\ \vdots \\ 0_n \end{Bmatrix} \tag{4}$$

where f_i and α_i are respectively the amplitude and the phase of the harmonic component acting on the i^{th} dof.

The total system mechanical energy in the presence of a harmonic force is given by the sum of the elastic and kinetic energy contributions:

$$E(t) = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}(t)\mathbf{K}\boldsymbol{x}(t) + \frac{1}{2}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\mathbf{M}\dot{\boldsymbol{x}}(t) \tag{5}$$

In order to evaluate the contribution of each interior mode to the system energy the set of physical coordinates in Eq. (5) is transformed by means of the CB basis, defined in Eq. (2):

$$E(t) = \frac{1}{2}\boldsymbol{y}^{\mathrm{T}}(t)\mathbf{H}^{\mathrm{T}}\mathbf{K}\mathbf{H}\boldsymbol{y}(t) + \frac{1}{2}\dot{\boldsymbol{y}}^{\mathrm{T}}(t)\mathbf{H}^{\mathrm{T}}\mathbf{M}\mathbf{H}\dot{\boldsymbol{y}}(t) \tag{6}$$

Equation (6) introduces matrices $\mathbf{K}^{\mathrm{CB}} = \mathbf{H}^{\mathrm{T}}\mathbf{K}\mathbf{H}$ and $\mathbf{M}^{\mathrm{CB}} = \mathbf{H}^{\mathrm{T}}\mathbf{M}\mathbf{H}$, which are the stiffness and the mass matrices in the CB basis. If, without lack of generality, matrices $\mathbf{M}$ and $\mathbf{K}$ are partitioned in accordance with the definition of the external and internal coordinates, namely:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{ee} & \mathbf{M}_{ei} \\ \mathbf{M}_{ie} & \mathbf{M}_{ii} \end{bmatrix}; \qquad \mathbf{K} = \begin{bmatrix} \mathbf{K}_{ee} & \mathbf{K}_{ei} \\ \mathbf{K}_{ie} & \mathbf{K}_{ii} \end{bmatrix} \tag{7}$$

and the modal matrix $\boldsymbol{\Phi}$ is normalized with respect to the mass matrix of the internal dof subsystem $\mathbf{M}_{ii}$, the following expressions are obtained for $\mathbf{K}^{CB}$ and $\mathbf{M}^{CB}$ [6]:

$$\begin{aligned} \mathbf{K}^{CB} &= \mathbf{H}^{T}\mathbf{K}\mathbf{H} = \begin{bmatrix} \mathbf{K}_{ee} + \mathbf{K}_{ei}\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega} \end{bmatrix} \\ \mathbf{M}^{CB} &= \mathbf{H}^{T}\mathbf{M}\mathbf{H} = \begin{bmatrix} \mathbf{M}_{ee} + \mathbf{B}^{T}\mathbf{M}_{ei}^{T} + \mathbf{M}_{ei}\mathbf{B} + \mathbf{B}^{T}\mathbf{M}_{ii}\mathbf{B} & \left(\mathbf{M}_{ei} + \mathbf{B}^{T}\mathbf{M}_{ii}\right)\boldsymbol{\Phi} \\ \boldsymbol{\Phi}^{T}\left(\mathbf{M}_{ei}^{T} + \mathbf{M}_{ii}\mathbf{B}\right) & \mathbf{I}_{s} \end{bmatrix} \end{aligned} \tag{8}$$

where $\boldsymbol{\Omega} \in \mathbb{R}^{s \times s}$ is the diagonal matrix of the squared angular eigenfrequencies associated to the interior modes.

In order to provide a clearer expression of the equations, the submatrices of the matrices in Eq. (8) will hereafter be referred to with the following compact notation:

$$\mathbf{K}^{CB} = \begin{bmatrix} \mathbf{K}_{ee}^{CB} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega} \end{bmatrix}; \qquad \mathbf{M}^{CB} = \begin{bmatrix} \mathbf{M}_{ee}^{CB} & \mathbf{M}_{ei}^{CB} \\ \mathbf{M}_{ie}^{CB} & \mathbf{I}_{s} \end{bmatrix} \tag{9}$$

The system steady-state response $\boldsymbol{y}$ to the harmonic force is itself harmonic:

$$\boldsymbol{y}(t) = \mathbf{H}^{-1}\boldsymbol{x}(t) = \begin{Bmatrix} \boldsymbol{x}_{e}(t) \\ \boldsymbol{\eta}(t) \end{Bmatrix} = \begin{Bmatrix} x_{e_1}\cos(\omega t + \gamma_1) \\ \vdots \\ x_{e_m}\cos(\omega t + \gamma_m) \\ \eta_1\cos(\omega t + \beta_1) \\ \vdots \\ \eta_s\cos(\omega t + \beta_{s1}) \end{Bmatrix} \tag{10}$$

where x_{e_i} $(i = 1, \dots, m)$ and η_j $(j = 1, \dots, s)$ are the amplitudes of, respectively, each master dof and each interior modal coordinate, while γ_i and β_j are the phases.

Introducing Eq. (10) in Eq. (6), the system mechanical energy (kinetic and potential elastic energy) can be rewritten by separating the contributions of the master dofs $\boldsymbol{x}_e$ and the interior modal coordinates $\boldsymbol{\eta}$:

$$\begin{aligned} E(t) = \frac{1}{2}\Big(&\boldsymbol{x}_e^{T}(t)\mathbf{K}_{ee}^{CB}\boldsymbol{x}_e(t) \\ &+ \dot{\boldsymbol{x}}_e^{T}(t)\mathbf{M}_{ee}^{CB}\dot{\boldsymbol{x}}_e(t) + 2\dot{\boldsymbol{x}}_e^{T}(t)\mathbf{M}_{ei}^{CB}\dot{\boldsymbol{\eta}}(t) + \boldsymbol{\eta}^{T}(t)\boldsymbol{\Omega}\boldsymbol{\eta}(t) + \dot{\boldsymbol{\eta}}^{T}(t)\dot{\boldsymbol{\eta}}(t)\Big) \end{aligned} \tag{11}$$

Equation (11) clearly highlights which are the contributions of the interior modes to system energy computation. In particular, the first two terms in Eq. (11) represent the energy contribution of the *m*-dimensional external dof subsystem (represented through $\mathbf{K}_{ee}^{CB}$ and $\mathbf{M}_{ee}^{CB}$), and therefore, they can be discarded in the evaluation of the energy contribution of each interior mode. In contrast, the other terms in Eq. (11) represent the contribution of the *s*-dimensional interior dof subsystem to the mechanical energy: they are a purely kinetic term ($\dot{\boldsymbol{\eta}}^{T}\dot{\boldsymbol{\eta}}$) an elastic term ($\boldsymbol{\eta}^{T}\boldsymbol{\Omega}\boldsymbol{\eta}$), and a term due to the inertial coupling between $\boldsymbol{x}_e$ and $\boldsymbol{\eta}$ ($2\dot{\boldsymbol{x}}_e^{T}\mathbf{M}_{ei}^{CB}\boldsymbol{\Phi}\boldsymbol{\eta}$). In order to evaluate the contribution of each interior mode to the system dynamics, let us consider just the summation of such terms of Eq. (11), referred to as E_s:

$$E_s(t) = \frac{1}{2}\Big(2\dot{\boldsymbol{x}}_e^{T}(t)\mathbf{M}_{ei}^{CB}\boldsymbol{\eta}(t) + \boldsymbol{\eta}^{T}(t)\boldsymbol{\Omega}\boldsymbol{\eta}(t) + \dot{\boldsymbol{\eta}}^{T}(t)\dot{\boldsymbol{\eta}}(t)\Big) \tag{12}$$

Finally, let us consider the mean value of $E_s(t)$ on a period $\tau = 2\pi/\omega$:

$$\overline{\mathrm{E}}_{\mathrm{s}} = \frac{1}{\tau}\int_{0}^{\tau} E_s(t)dt = \frac{1}{2}\sum_{\varsigma=1}^{s}\left(\frac{\left(\omega_\varsigma^2+\omega^2\right)\eta_\varsigma^2}{2} + \omega^2\sum_{\mu=1}^{\mathrm{m}} \mathrm{x}_{\mathrm{e}_\mu}^{\mathrm{T}}\mathrm{M}_{\mathrm{ei}_{\mu,\varsigma}}^{\mathrm{CB}}\cos\left(\gamma_\mu-\beta_\varsigma\right)\eta_\varsigma\right) \tag{13}$$

This time-independent scalar quantity is a meaningful and compact measure of the contribution of the interior modes to the computation of the mechanical energy stored in a period. In particular, from Eq. (13), it is evident that the contribution of the ς^{th} interior mode to the system mean energy $\overline{\mathrm{E}}_{\mathrm{s}}$ is weighted through the scalar coefficients Γ_ς,:

$$\Gamma_\varsigma = \left|\frac{\left(\omega_\varsigma^2+\omega^2\right)\eta_\varsigma^2}{2} + \omega^2\sum_{\mu=1}^{\mathrm{m}} \mathrm{x}_{\mathrm{e}_\mu}^{\mathrm{T}}\mathrm{M}_{\mathrm{ei}_{\mu,\varsigma}}^{\mathrm{CB}}\cos\left(\gamma_\mu-\beta_\varsigma\right)\eta_\varsigma\right| \qquad \varsigma = 1,\dots,\mathrm{s} \tag{14}$$

The larger the value of Γ_ς, the more the ς^{th} interior mode is relevant to the computation of $\overline{\mathrm{E}}_{\mathrm{s}}$ and thus of $\overline{\mathrm{E}}$, and hence, the more it contributes to the system response in the presence of the harmonic force $\boldsymbol{f}$ defined through Eq.(4). Therefore, the interior modes can be ranked with descending values of Γ_ς, and progressively included in the reduced model until a desired degree of accuracy of the model is achieved.

3. METHOD APPLICATION

3.1. Evaluation indices

In order to be useful in practice, for example at system design early stages, a reduced model must provide an accurate representation of the system forced response both in terms of spatial distribution and amplitude. The accuracy of the reduced model is therefore evaluated by means of two indexes. The first one is the modal assurance criterion (MAC) between the vectors of the forced response computed at a certain time instant $(\bar{t})$ in all the nodes by the full-order model $(\boldsymbol{x})$ and by the reduced-order one $\left(\tilde{\boldsymbol{x}} = \tilde{\mathbf{H}}\tilde{\boldsymbol{y}}\right)$:

$$\mathrm{MAC} = \frac{\left(\boldsymbol{x}^{\mathrm{T}}(\bar{t})\tilde{\boldsymbol{x}}(\bar{t})\right)^2}{\left(\boldsymbol{x}^{\mathrm{T}}(\bar{t})\boldsymbol{x}(\bar{t})\right)\left(\tilde{\boldsymbol{x}}^{\mathrm{T}}(\bar{t})\tilde{\boldsymbol{x}}(\bar{t})\right)} \tag{15}$$

The second index is the relative gain error at the frequency of interest, i.e. the relative percentage error between the amplitude of the forced response computed by the full-order model and by the reduced-order one, evaluated at the frequency of interest for one arbitrary master dof (denoted through index *j*):

$$\varepsilon(\omega) = \frac{\mathrm{x}_j(\omega) - \tilde{\mathrm{x}}_j(\omega)}{\mathrm{x}_j(\omega)} \cdot 100 \tag{16}$$

Obviously the target value for the first index is one, while for the second is zero.

3.2. Test case: Vibratory feeder

3.2.1. System description

The vibrating system adopted for method validation is the one represented in Figure 1. It represents a linear vibratory feeder, of the kind usually employed in the packaging industries for conveying small components or products. Basically, a number of four-dof Euler-Bernoulli beam finite elements are employed to model both the feeder tray (along which conveyed products move), and the support beam, which is connected to the feeder tray by means of six linear spring and to the ground through two elastic supports modeled by linear springs too. The three concentrated and suspended masses in Figure 1 represent the external electromagnetic actuators, fixed to the feeder tray which excite it. Such a system model leads to 39 degrees of freedom (dofs). In particular, the six dofs on which the actuation forces are applied are considered the master dofs. As far as such forces are concerned, it is

supposed that each of the three actuators generates a harmonic force at 50 Hz on the tray and an identical reaction force on its suspended mass. All actuation forces are supposed in-phase and identical in amplitude. Such an hypothesis is reasonable since electromagnetic exciter are usually driven by identical and in-phase harmonic currents.

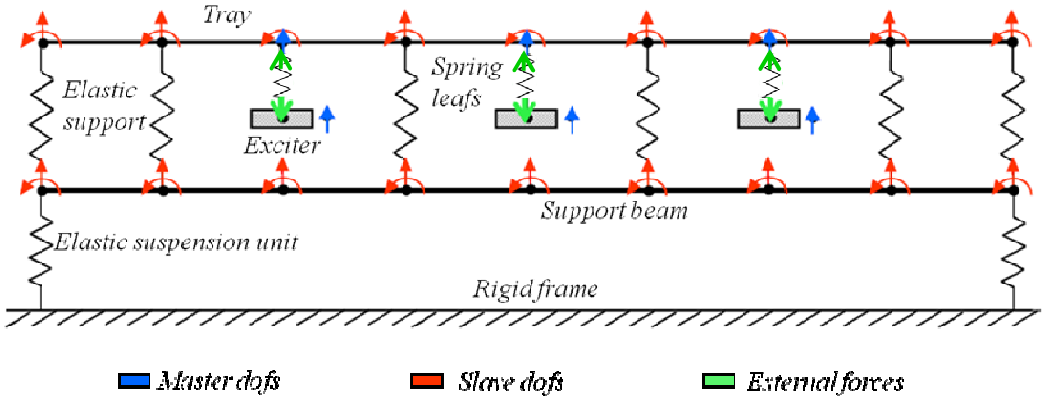

Figure 1. Finite element model of the vibratory feeder investigated.

3.2.2. Method application and results

The application of the proposed ranking method and of the benchmark ones leads to the results summarized in Figure 2, where the two evaluation parameters defined above, are plotted as functions of the number of interior modes used in the reduced order representation. Clearly, for each method, interior modes have been added based on the ranking derived using the method itself.

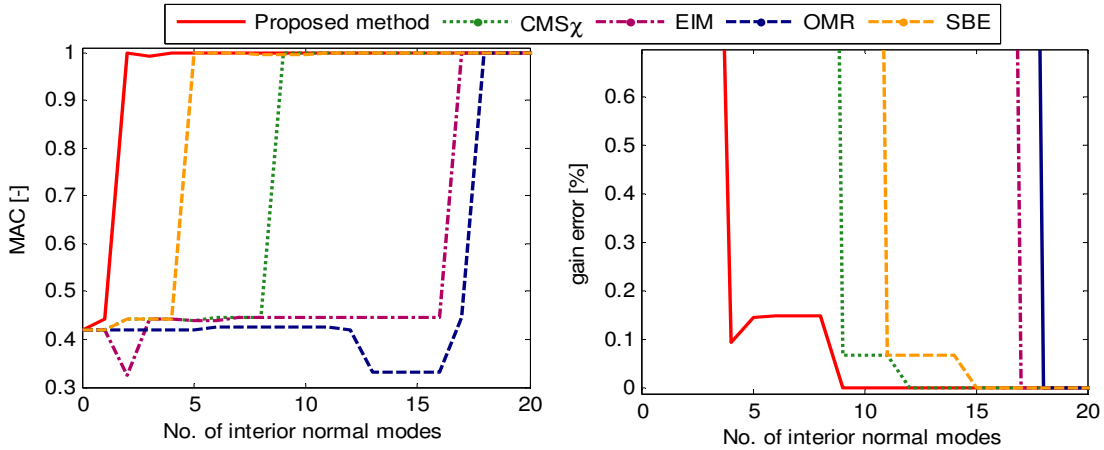

Figure 2. MAC (left) and ε_g (right) vs number of interior modes.

The capability of the proposed approach to ensure accuracy through a minimum set of properly selected interior modes is clearly proved by the results obtained. Indeed, the convergence of the proposed method to the ideal results (i.e. MAC $= 1$ and $\varepsilon_g(\omega) = 0$) outperforms the ones of the four benchmark methods. For instance, by supposing that reasonable accuracy thresholds of 0.999 for MAC and of 0.2% for $\varepsilon_g(\omega)$ are prescribed, a 10-dimensional model (with 6 master dofs and just 4 out of the 33 interior modes) is adequate if the retained modes are selected through the method proposed here. This leads to a model order reduction ratio of 74.3%, to be intended as the ratio between the number of dofs neglected in the reduced model and the order of the full model. Conversely, much higher model dimensions are needed to achieve the same accuracy by the other ranking the modes with other methods available in literature. Compared performances are shown in Table 1.

Table 1. Number of interior modes necessary for each method to obtain a MAC ≥ 0.999 and a $\varepsilon_g \leq 0.2\%$

METHOD	NUMBER OF INTERIOR MODES	REDUCED MODEL DIMENSIONS	ORDER REDUCTION RATIO
Proposed Method	4	10	74.3%
Component Mode Synthesis χ	9	15	61.5%
Effective Interface Mass	17	23	41.0%
Optimal Modal Reduction	18	24	38.4%
Sorting Based on Eigenfrequency	11	17	56.4%

4. CONCLUSIONS

The results obtained highlight that one of the most critical issue to be tackled in the application of the CB method is the proper selection of the interior modes to be retained in accordance to their contribution to the system dynamics. The method proposed in this paper provides an effective solution to such an issue, by introducing energy-based analytical scalar coefficients which have been proved useful to rank the interior modes. The method can be adopted whenever the frequency response of a system excited with an harmonic force (whose frequency and spatial distribution are known) must be accurately represented through a reduced order model. Such a need is quite common in the design and optimization of vibrating systems such as vibratory feeder, sieves, or, more in general, of vibration generators.

The theory proposed, which is general, is here applied to a vibratory feeder. The method effectiveness is corroborated through the comparison between its outcome and the ones provided by other ranking techniques available in literature. It is proved that the proposed method achieves the desired levels of accuracy with a minimal set of interior modes.

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REFERENCES

[1] Vidoni, R., Gasparetto, A. & Giovagnoni, M., (2013), Design and implementation of an ERLS-based 3-D dynamic formulation for flexible-link robots. *Robot. Comput. Integrated Manuf.,* 29(2), 273-282.

[2] Caracciolo, R., Richiedei, D. & Trevisani, A., (2006), Design and experimental validation of piecewise-linear state observers for flexible link mechanisms. *Meccanica,* 41(6), 623-637.

[3] Ouyang, H., Richiedei, D., Trevisani, A. & Zanardo, G., (2012), Eigenstructure assignment in undamped vibrating systems: A convex-constrained modification method based on receptances. *Mechanical Systems and Signal Processing,* 27397-409.

[4] Richiedei, D., Trevisani, A. & Zanardo, G., (2011), A constrained convex approach to modal design optimization of vibrating systems. *Journal of Mechanical Design,* 133(6), 061011.

[5] Pumhoessel, T., Hehenberger, P. & Zeman, K., (2012), Model reduction of a parametrically excited drivetrain. In: *Proc. ASME 2012 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 1025-1034).

[6] Bampton, M. & Craig, R., (1968), Coupling of substructures for dynamic analyses. *AIAA J.,* 6(7), 1313-1319.

[7] Hintz, R. M., (1975), Analytical methods in component modal synthesis. *AIAA J.,* 13(8), 1007-1016.

[8] Liao, B., Bai, Z. & Gao, W., (2007), The important modes of subsystems: A moment-matching approach. *Int J Numer Methods Eng,* 70(13), 1581-1597.

[9] Kammer, D. C. & Triller, M. J., (1996), Selection of component modes for Craig-Bampton substructure representations. *Journal of Vibration and Acoustics,* 118(2), 264-270.

[10] Kammer, D. & Triller, M., (1994), Ranking the dynamic importance of fixed interface modes using a generalization of effective mass. *Modal Analysis: The International Journal of Analytical and Experimental Modal Analysis,* 9(2), 77-98.

[11] Palomba, I., Richiedei, D. & Trevisani, A., (2014), A ranking method for the selection of the interior modes of reduced order resonant system models. In: *Proc. ASME 2014 12th Biennial Conference on Engineering Systems Design and Analysis* Copenhagen, Denmark, 2014, June 25-27.

[12] Barbone, P. E., Givoli, D. & Patlashenko, I., (2003), Optimal modal reduction of vibrating substructures. *Int J Numer Methods Eng,* 57(3), 341-369.

[13] Givoli, D., Barbone, P. E. & Patlashenko, I., (2004), Which are the important modes of a subsystem? *Int J Numer Methods Eng,* 59(12), 1657-1678.